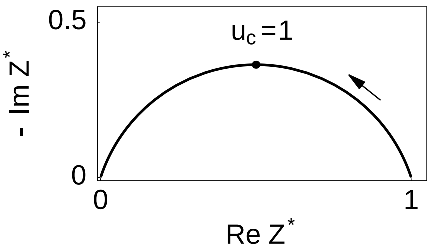
Equivalent electrical circuits containing CPEs



ER@SE

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Contents

Chapter 1

Circuits containing one CPE

1.1 Constant Phase Element (CPE), symbol Q

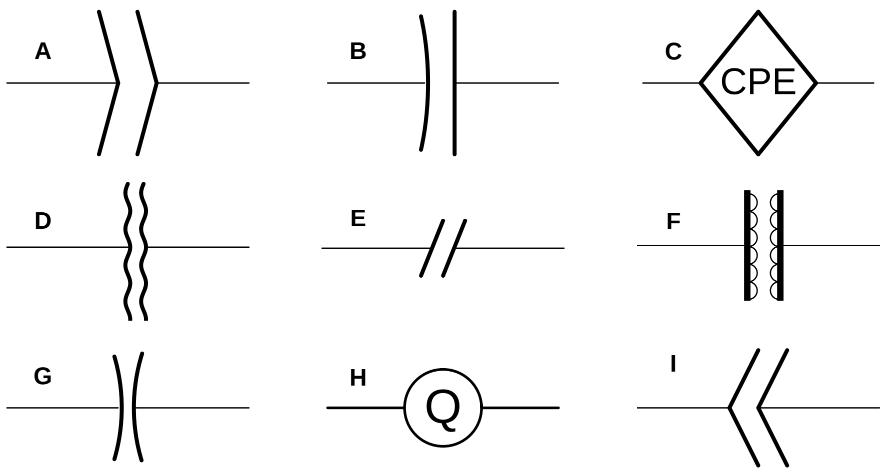

Figure 1.1: Some CPE symbols, taken from A: [5], B: [8], C: [11], D: [1], E: [3], F: [6], G: [7], H: [9], I: [4].

$$Z = \frac{1}{Q\,(\mathrm{i}\,\omega)^{\alpha}}\ ,\ \mathrm{Re}\,Z = \frac{c_{\alpha}}{Q\,\omega^{\alpha}}\ ,\ \mathrm{Im}\,Z = -\frac{s_{\alpha}}{Q\,\omega^{\alpha}}$$

$$c_{\alpha} = \cos(\frac{\pi\,\alpha}{2})\ ,\ s_{\alpha} = \sin(\frac{\pi\,\alpha}{2})$$

$$|Z| = \frac{1}{Q\,\omega^{\alpha}}\ ,\ \phi_{Z} = -\frac{\pi\,\alpha}{2}$$

The Q unit depends on α: $\mathrm{u}_{Q} = \mathrm{F\,cm^{-2}\,s^{\alpha-1}}$.

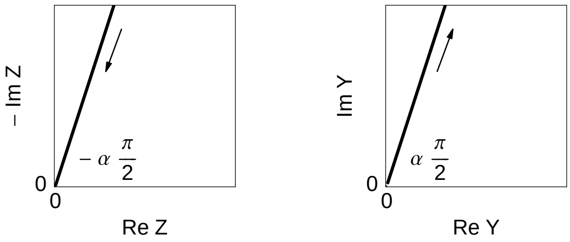

Figure 1.2: Nyquist diagram of the impedance and admittance for the CPE element plotted for $\alpha = 0.8$. The arrows always indicate the increasing frequency direction.

1.2 Circuit (R+Q)

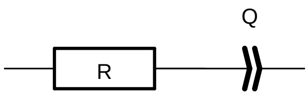

Figure 1.3: Circuit (R+Q).

1.2.1 Impedance

$$Z(\omega) = R + \frac{1}{Q\,(\mathrm{i}\,\omega)^{\alpha}}\ ,\ \mathrm{Re}\,Z = R + \frac{c_{\alpha}}{Q\,\omega^{\alpha}}\ ,\ \mathrm{Im}\,Z = -\frac{s_{\alpha}}{Q\,\omega^{\alpha}}$$

1.2.2 Reduced impedance

$$Z^{*}(\omega) = \frac{Z(\omega)}{R} = 1 + \frac{1}{\tau\,(\mathrm{i}\,\omega)^{\alpha}}\ ,\ \tau = R\,Q$$

The τ unit depends on α: $\mathrm{u}_{\tau} = \mathrm{s}^{\alpha}$.

$$Z^{*}(u) = 1 + \frac{1}{(\mathrm{i}\,u)^{\alpha}},\ u = \omega\,\tau^{1/\alpha}$$

1.3 Circuit (R/Q)

1.3.1 Impedance

$$Z(\omega) = \frac{R}{1 + \tau\,(\mathrm{i}\,\omega)^{\alpha}}\ ;\ \tau = R\,Q$$

$$\mathrm{Re}\,Z(\omega) = \frac{R\,(1 + \tau\,\omega^{\alpha}\,c_{\alpha})}{1 + \tau^{2}\,\omega^{2\,\alpha} + 2\,\tau\,\omega^{\alpha}\,c_{\alpha}}\ ;\ \mathrm{Im}\,Z(\omega) = -\frac{R\,\tau\,\omega^{\alpha}\,s_{\alpha}}{1 + \tau^{2}\,\omega^{2\,\alpha} + 2\,\tau\,\omega^{\alpha}\,c_{\alpha}}$$

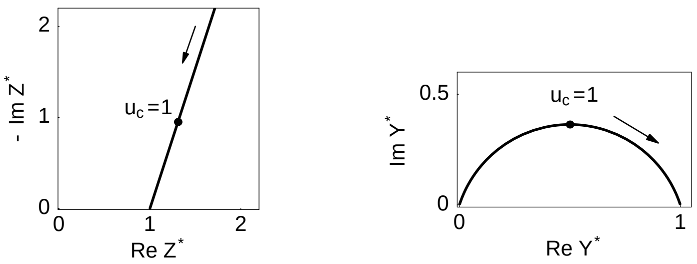

Figure 1.4: Nyquist diagram of the reduced impedance and admittance ($Y^* = R\,Y$) for the (R+Q) circuit plotted for $\alpha = 0.8$.

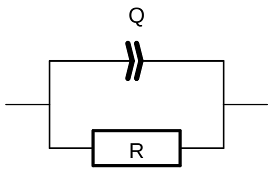

Figure 1.5: Circuit (R/Q).

1.3.2 Reduced impedance

$$Z^*(\omega) = \frac{Z(\omega)}{R} = \frac{1}{1+\tau\,(\mathrm{i}\,\omega)^\alpha}\ ;\ \tau = R\,Q$$

$$\mathrm{Re}\ Z^*(\omega) = \frac{1+\tau\,\omega^\alpha\,c_\alpha}{1+\tau^2\,\omega^{2\,\alpha}+2\,\tau\,\omega^\alpha\,c_\alpha}\ ;\ \mathrm{Im}\ Z^*(\omega) = -\frac{\tau\,\omega^\alpha\,s_\alpha}{1+\tau^2\,\omega^{2\,\alpha}+2\,\tau\,\omega^\alpha\,c_\alpha}$$

$$\frac{\mathrm{dIm}\ Z^*(\omega)}{\mathrm{d}\omega} = \frac{\alpha\,\tau\,\omega^{-1+\alpha}\,\left(-1+\tau^2\,\omega^{2\,\alpha}\right)\,s_\alpha}{\left(1+\tau^2\,\omega^{2\,\alpha}+2\,\tau\,\omega^\alpha\,c_\alpha\right)^2} = 0 \Rightarrow \omega_c^\alpha = 1/\tau\ [2]$$

$$\mathrm{Re}\ Z^*(\omega_c) = 1/2\ ,\ \mathrm{Im}\ Z^*(\omega_c) = -\frac{s_\alpha}{2\,(1+c_\alpha)}$$

$$\alpha = \frac{2}{\pi}\ \arccos\left(-1+\frac{2}{1+4\,\mathrm{Im}\ Z^*(\omega_c)^2}\right)$$

$$Z^*(u) = \frac{1}{1+(\mathrm{i}\,u)^\alpha},\ u = \omega\,\tau^{1/\alpha}$$

1.4 Circuit (R/Q)+(R/Q)+ .. (Voigt)

$$Z(\omega) = \sum_{i=1}^{n_{\mathrm{RQ}}} \frac{R_i}{1+\tau_i\,(\mathrm{i}\,\omega)^{\alpha_i}}\ ;\ \tau_i = R_i\,Q_i$$

$$\mathrm{Re}\ Z(\omega) = \sum_{i=1}^{n_{\mathrm{RQ}}} \frac{R_i\ (1+\tau_i\,\omega^{\alpha_i}\,c_{\alpha i})}{1+\tau_i^2\,\omega^{2\,\alpha_i}+2\,\tau_i\,\omega^{\alpha_i}\,c_{\alpha i})}$$

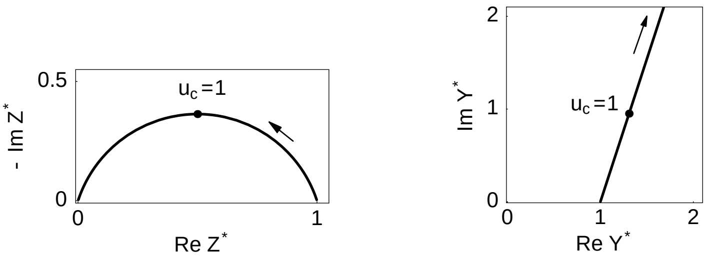

Figure 1.6: Nyquist diagram of the reduced impedance (depressed semi-circular arc [10]) and admittance ($Y^* = R\,Y$) for the (R/Q) circuit plotted for $\alpha = 0.8$.

$$\mathrm{Im}\; Z(\omega) = -\sum_{i=1}^{n_{\mathrm{RQ}}} \frac{R_i\,\tau_i\,\omega^{\alpha_i}\;s_{\alpha i}}{1+\tau_i^2\,\omega^{2\,\alpha_i}+2\,\tau_i\,\omega^{\alpha_i}\,c_{\alpha i}}$$

Chapter 2

Circuits made of one R and two CPEs

2.1 Circuit $((\mathbf{R}_1/\mathbf{Q}_1)+\mathbf{Q}_2)$

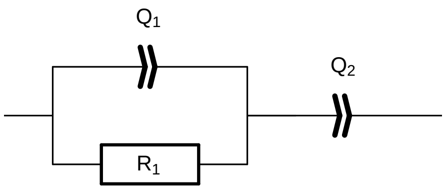

Figure 2.1: Circuit ((R$_1$/Q$_1$)+Q$_2$).

2.1.1 Impedance

$$Z(\omega)=\frac{1}{(\mathrm{i}\,\omega)^{\alpha_2}\,Q_2}+\frac{R_1}{(\mathrm{i}\,\omega)^{\alpha_1}\,Q_1\,\left(\frac{1}{(\mathrm{i}\,\omega)^{\alpha_1}\,Q_1}+R_1\right)}$$

$$Z(\omega)=\frac{R_1\,\left(1+(\mathrm{i}\,\omega)^{\alpha_1}\,\tau_1+(\mathrm{i}\,\omega)^{\alpha_2}\,\tau_2\right)}{(\mathrm{i}\,\omega)^{\alpha_2}\,\left(1+(\mathrm{i}\,\omega)^{\alpha_1}\,\tau_1\right)\,\tau_2}\;,\;\tau_1=R_1\,Q_1\;,\;\tau_2=R_1\,Q_2$$

2.1.2 Reduced impedance

$$Z^*(\omega)=\frac{Z(\omega)}{R}=\frac{1+(\mathrm{i}\,\omega)^{\alpha_1}\,\tau_1+(\mathrm{i}\,\omega)^{\alpha_2}\,\tau_2}{(\mathrm{i}\,\omega)^{\alpha_2}\,\left(1+(\mathrm{i}\,\omega)^{\alpha_1}\,\tau_1\right)\,\tau_2}$$

$$\mathrm{Re}\;Z^*(\omega)=\frac{1+\omega^{\alpha_1}\,c_{\alpha1}\,\tau_1}{1+\omega^{\alpha_1}\,\tau_1\,\left(2\,c_{\alpha1}+\omega^{\alpha_1}\,\tau_1\right)}+\frac{c_{\alpha2}}{\omega^{\alpha_2}\,\tau_2}$$

$$s_{\alpha1}=\sin(\frac{\pi\,\alpha_1}{2})\;,\;s_{\alpha2}=\sin(\frac{\pi\,\alpha_2}{2})\;,\;c_{\alpha1}=\cos(\frac{\pi\,\alpha_1}{2})\;,\;c_{\alpha2}=\cos(\frac{\pi\,\alpha_2}{2})$$

$$\text{Im}\, Z^*(\omega) = -\frac{\omega^{\alpha_1}\, s_{\alpha 1}\, \tau_1}{1 + \omega^{\alpha_1}\, \tau_1\, \left(2\, c_{\alpha 1} + \omega^{\alpha_1}\, \tau_1\right)} - \frac{s_{\alpha 2}}{\omega^{\alpha_2}\, \tau_2}$$

2.2 Circuit $((\mathbf{R}_1 + \mathbf{Q}_1)/\mathbf{Q}_2)$

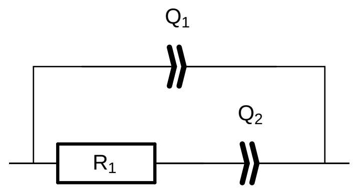

Figure 2.2: Circuit $((R_1{+}Q_2)/Q_1)$.

$$Z(\omega) = \frac{\frac{1}{(\mathrm{i}\,\omega)^{\alpha_2}\, Q_2} + R_1}{(\mathrm{i}\,\omega)^{\alpha_1}\, Q_1\, \left(\frac{1}{(\mathrm{i}\,\omega)^{\alpha_1}\, Q_1} + \frac{1}{(\mathrm{i}\,\omega)^{\alpha_2}\, Q_2} + R_1\right)}$$

$$Z(\omega) = \frac{1 + \tau\, (\mathrm{i}\,\omega)^{\alpha_2}}{(\mathrm{i}\,\omega)^{\alpha_1}\; Q_1 + (\mathrm{i}\,\omega)^{\alpha_2}\; Q_2 + \tau\, (\mathrm{i}\,\omega)^{\alpha_1+\alpha_2}\, Q_1}\ ,\ \tau = R_1\, Q_2$$

$\text{Re}\, Z(\omega) =$

$$\left(\omega^{\alpha_1}\, c_{\alpha 1}\, \left(1 + \tau^2\, \omega^{2\,\alpha_2} + 2\,\tau\,\omega^{\alpha_2}\, c_{\alpha 2}\right)\, Q_1 + \omega^{\alpha_2}\, \left(\tau\,\omega^{\alpha_2} + c_{\alpha 2}\right)\, Q_2\right) /$$
$$\left(\omega^{2\,\alpha_1}\, \left(1 + \tau^2\, \omega^{2\,\alpha_2} + 2\,\tau\,\omega^{\alpha_2}\, c_{\alpha 2}\right)\, {Q_1}^2 + 2\,\omega^{\alpha_1+\alpha_2}\, \left(\tau\,\omega^{\alpha_2}\, c_{\alpha 1} + c_{\alpha 1 m \alpha 2}\right)\, Q_1\, Q_2 + \omega^{2\,\alpha_2}\, {Q_2}^2\right)$$

$$c_{\alpha 1 m \alpha 2} = \cos\left(\frac{\pi\, (\alpha_1 - \alpha_2)}{2}\right)$$

$\text{Im}\, Z(\omega) =$

$$\left(-\omega^{\alpha_1}\, \left(1 + \tau^2\, \omega^{2\,\alpha_2} + 2\,\tau\,\omega^{\alpha_2}\, c_{\alpha 2}\right)\, Q_1\, s_{\alpha 1} - \omega^{\alpha_2}\, Q_2\, s_{\alpha 2}\right) /$$
$$\left(\omega^{2\,\alpha_1}\, \left(1 + \tau^2\, \omega^{2\,\alpha_2} + 2\,\tau\,\omega^{\alpha_2}\, \alpha 2\right)\, {Q_1}^2 + 2\,\omega^{\alpha_1+\alpha_2}\, \left(\tau\,\omega^{\alpha_2}\, c_{\alpha 1} + c_{\alpha 1 m \alpha 2}\right)\, Q_1\, Q_2 + \omega^{2\,\alpha_2}\, {Q_2}^2\right)$$

Chapter 3

Circuits made of two Rs and two CPEs

3.1 Circuit $((R_1/Q_1)+(R_2/Q_2))$

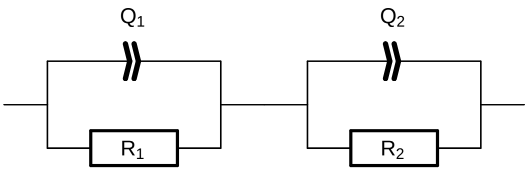

Figure 3.1: Circuit $((R_1/Q_1)+(R_2/Q_2))$.

$$Z(\omega) = \frac{1}{(\mathrm{i}\,\omega)^{\alpha_1}\,Q_1 + \dfrac{1}{R_1}} + \frac{1}{(\mathrm{i}\,\omega)^{\alpha_2}\,Q_2 + \dfrac{1}{R_2}}$$

$$Z(\omega) = \frac{R_1}{1+(\mathrm{i}\,\omega)^{\alpha_1}\,\tau_1} + \frac{R_2}{1+(\mathrm{i}\,\omega)^{\alpha_2}\,\tau_2}\ ,\ \tau_1 = R_1\,Q_1\ ,\ \tau_2 = R_2\,Q_2$$

$$Z(\omega) = \frac{R_1 + R_2 + (\mathrm{i}\,\omega)^{\alpha_1}\,R_2\,\tau_1 + (\mathrm{i}\,\omega)^{\alpha_2}\,R_1\,\tau_2}{\left(1+(\mathrm{i}\,\omega)^{\alpha_1}\,\tau_1\right)\,\left(1+(\mathrm{i}\,\omega)^{\alpha_2}\,\tau_2\right)}$$

$$\mathrm{Re}\,Z(\omega) = \frac{R_1\,\left(1+\omega^{\alpha_1}\,c_{\alpha 1}\,\tau_1\right)}{1+\omega^{\alpha_1}\,\tau_1\,\left(2\,c_{\alpha 1}+\omega^{\alpha_1}\,\tau_1\right)} + \frac{R_2\,\left(1+\omega^{\alpha_2}\,c_{\alpha 2}\,\tau_2\right)}{1+\omega^{\alpha_2}\,\tau_2\,\left(2\,c_{\alpha 2}+\omega^{\alpha_2}\,\tau_2\right)}$$

$$\mathrm{Im}\,Z(\omega) = -\frac{\omega^{\alpha_1}\,R_1\,s_{\alpha 1}\,\tau_1}{1+\omega^{\alpha_1}\,\tau_1\,\left(2\,c_{\alpha 1}+\omega^{\alpha_1}\,\tau_1\right)} - \frac{\omega^{\alpha_2}\,R_2\,s_{\alpha 2}\,\tau_2}{1+\omega^{\alpha_2}\,\tau_2\,\left(2\,c_{\alpha 2}+\omega^{\alpha_2}\,\tau_2\right)}$$

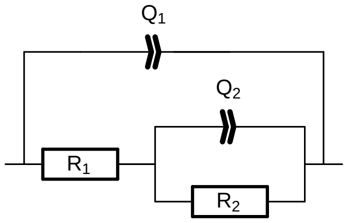

Figure 3.2: Circuit $((R_1+(R_2/Q_2))/Q_1)$.

3.2 Circuit $((\mathrm{R}_1+(\mathrm{R}_2/\mathrm{Q}_2))/\mathrm{Q}_1)$

$$Z(\omega) = \frac{1}{(\mathrm{i}\,\omega)^{\alpha_1}\,Q_1 + \cfrac{1}{R_1 + \cfrac{1}{(\mathrm{i}\,\omega)^{\alpha_2}\,Q_2 + \cfrac{1}{R_2}}}}$$

$$Z(\omega) = \frac{R_1 + R_2 + (\mathrm{i}\,\omega)^{\alpha_2}\,Q_2\,R_1\,R_2}{1 + (\mathrm{i}\,\omega)^{\alpha_1}\,Q_1\,(R_1+R_2) + (\mathrm{i}\,\omega)^{\alpha_2}\,Q_2\,R_2 + (\mathrm{i}\,\omega)^{\alpha_1+\alpha_2}\,Q_1\,Q_2\,R_1\,R_2}$$

$$\begin{gathered}\mathrm{Re}\,Z(\omega) = \big(R_1 + R_2 + \omega^{2\,\alpha_2}\,{Q_2}^2\,R_1\,\left(1+\omega^{\alpha_1}\,C_{\alpha 1}\,Q_1\,R_1\right)\,{R_2}^2 + \\ \omega^{\alpha_1}\,C_{\alpha 1}\,Q_1\,(R_1+R_2)^2 + \omega^{\alpha_2}\,C_{\alpha 2}\,Q_2\,R_2\,\left(R_2 + 2\,R_1\,\left(1+\omega^{\alpha_1}\,C_{\alpha 1}\,Q_1\,\left(R_1+R_2\right)\right)\right)\big)\,/ \\ \big(1 + \omega^{2\,\alpha_2}\,{Q_2}^2\,\left(1+\omega^{\alpha_1}\,Q_1\,R_1\,\left(2\,C_{\alpha 1} + \omega^{\alpha_1}\,Q_1\,R_1\right)\right)\,{R_2}^2 + \\ \omega^{\alpha_1}\,Q_1\,\left(R_1+R_2\right)\,\left(2\,C_{\alpha 1} + \omega^{\alpha_1}\,Q_1\,\left(R_1+R_2\right)\right) + 2\,\omega^{\alpha_2}\,Q_2\,R_2 \\ \times\left(C_{\alpha 2} + \omega^{\alpha_1}\,Q_1\,\left(C_{\alpha 1 m \alpha 2}\,R_2 + C_{\alpha 2}\,R_1\,\left(2\,C_{\alpha 1} + \omega^{\alpha_1}\,Q_1\,\left(R_1+R_2\right)\right)\right)\right)\big)\end{gathered}$$

$$c_{\alpha 1 m \alpha 2} = \cos\left(\frac{\pi\,\left(\alpha_1 - \alpha_2\right)}{2}\right)$$

$$\begin{gathered}\mathrm{Im}\,Z(\omega) = \Big(\omega^{\alpha_1}\,Q_1\,\Big(-\omega^{2\,\alpha_2}\,{Q_2}^2\,{R_1}^2\,{R_2}^2 - 2\,\omega^{\alpha_2}\,C_{\alpha 2}\,Q_2\,R_1\,R_2\,\left(R_1+R_2\right) - \\ \left(R_1+R_2\right)^2\Big)\,S_{\alpha 1} - \omega^{\alpha_2}\,Q_2\,{R_2}^2\,S_{\alpha 2}\Big)\,/ \\ \big(1 + \omega^{2\,\alpha_2}\,{Q_2}^2\,\left(1+\omega^{\alpha_1}\,Q_1\,R_1\,\left(2\,C_{\alpha 1} + \omega^{\alpha_1}\,Q_1\,R_1\right)\right)\,{R_2}^2 + \\ \omega^{\alpha_1}\,Q_1\,\left(R_1+R_2\right)\,\left(2\,C_{\alpha 1} + \omega^{\alpha_1}\,Q_1\,\left(R_1+R_2\right)\right) + 2\,\omega^{\alpha_2}\,Q_2\,R_2 \\ \times\left(C_{\alpha 2} + \omega^{\alpha_1}\,Q_1\,\left(C_{\alpha 1 m \alpha 2}\,R_2 + C_{\alpha 2}\,R_1\,\left(2\,C_{\alpha 1} + \omega^{\alpha_1}\,Q_1\,\left(R_1+R_2\right)\right)\right)\right)\big)\end{gathered}$$

3.3 Circuit $((\mathrm{Q}_1+(\mathrm{R}_2/\mathrm{Q}_2))/\mathrm{R}_1)$

$$Z(\omega) = \frac{1}{\cfrac{1}{R_1} + \cfrac{1}{\cfrac{1}{(\mathrm{i}\,\omega)^{\alpha_1}\,Q_1} + \cfrac{1}{(\mathrm{i}\,\omega)^{\alpha_2}\,Q_2 + \cfrac{1}{R_2}}}}$$

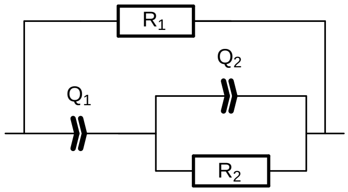

Figure 3.3: Circuit $((Q_1+(R_2/Q_2))/R_1)$.

$$Z(\omega) = \frac{R_1\;(1+(\mathrm{i}\,\omega)^{\alpha_1}\,Q_1\,R_2+(\mathrm{i}\,\omega)^{\alpha_2}\,Q_2\,R_2)}{1+(\mathrm{i}\,\omega)^{\alpha_1}\,Q_1\;(R_1+R_2)+(\mathrm{i}\,\omega)^{\alpha_2}\,Q_2\,R_2+(\mathrm{i}\,\omega)^{\alpha_1+\alpha_2}\,Q_1\,Q_2\,R_1\,R_2}$$

$$\begin{gathered}\mathrm{Re}\;\;Z(\omega) = \big(R_1\;\big(1+\omega^{\alpha_2}\,Q_2\,R_2\;(2\,C_{\alpha 2}+\omega^{\alpha_2}\,Q_2\,R_2)+\omega^{2\,\alpha_1}\,{Q_1}^2\,R_2\\ \times(R_2+R_1\;(1+\omega^{\alpha_2}\,C_{\alpha 2}\,Q_2\,R_2))+\omega^{\alpha_1}\,Q_1\;(2\,R_2\;(C_{\alpha 1}+\omega^{\alpha_2}\,C_{\alpha 1 m\alpha 2}\,Q_2\,R_2)+\\ C_{\alpha 1}\,R_1\;(1+\omega^{\alpha_2}\,Q_2\,R_2\;(2\,C_{\alpha 2}+\omega^{\alpha_2}\,Q_2\,R_2)))))\,/\\ \big(1+\omega^{2\,\alpha_2}\,{Q_2}^2\;(1+\omega^{\alpha_1}\,Q_1\,R_1\;(2\,C_{\alpha 1}+\omega^{\alpha_1}\,Q_1\,R_1))\;{R_2}^2+\\ \omega^{\alpha_1}\,Q_1\;(R_1+R_2)\;(2\,C_{\alpha 1}+\omega^{\alpha_1}\,Q_1\;(R_1+R_2))+\\ 2\,\omega^{\alpha_2}\,Q_2\,R_2\;(C_{\alpha 2}+\omega^{\alpha_1}\,Q_1\;(C_{\alpha 1 m\alpha 2}\,R_2+C_{\alpha 2}\,R_1\;(2\,C_{\alpha 1}+\omega^{\alpha_1}\,Q_1\;(R_1+R_2)))))\end{gathered}$$

$$\begin{gathered}\mathrm{Im}\;\;Z(\omega) = -\omega^{\alpha_1}\,Q_1\,{R_2}^2\;(S_{\alpha 1}+\omega^{\alpha_2}\,Q_2\,R_2\;((2\,C_{\alpha 2}+\omega^{\alpha_2}\,Q_2\,R_2)\;S_{\alpha 1}+\omega^{\alpha_1}\,Q_1\,R_2\,S_{\alpha 2}))\,/\\ \big(1+\omega^{2\,\alpha_2}\,{Q_2}^2\;(1+\omega^{\alpha_1}\,Q_1\,R_1\;(2\,C_{\alpha 1}+\omega^{\alpha_1}\,Q_1\,R_1))\;{R_2}^2+\\ \omega^{\alpha_1}\,Q_1\;(R_1+R_2)\;(2\,C_{\alpha 1}+\omega^{\alpha_1}\,Q_1\;(R_1+R_2))+\\ 2\,\omega^{\alpha_2}\,Q_2\,R_2\;(C_{\alpha 2}+\omega^{\alpha_1}\,Q_1\;(C_{\alpha 1 m\alpha 2}\,R_2+C_{\alpha 2}\,R_1\;(2\,C_{\alpha 1}+\omega^{\alpha_1}\,Q_1\;(R_1+R_2)))))\end{gathered}$$

$$Z(\omega) = \frac{R_1\;(1+\tau_1\,(\mathrm{i}\,\omega)^{\alpha_1}+\tau_2\,(\mathrm{i}\,\omega)^{\alpha_2})}{1+(1+R_1/R_2)\;\tau_1\,(\mathrm{i}\,\omega)^{\alpha_1}+\tau_2\,(\mathrm{i}\,\omega)^{\alpha_2}+\tau_1\,\tau_2\,(R_1/R_2)\,(\mathrm{i}\,\omega)^{\alpha_1+\alpha_2}}$$

$$\tau_1 = Q_1\,R_2\;,\;\;\tau_2 = Q_2\,R_2$$

3.4 Circuit $(((Q_2+R_2)/R_1)/Q_1)$

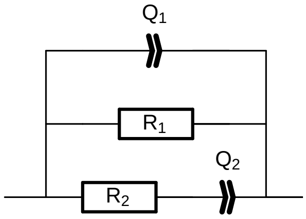

Figure 3.4: Circuit $(((Q_2+R_2)/R_1)/Q_1)$.

$$Z(\omega) = \frac{1}{(\mathrm{i}\,\omega)^{\alpha_1}\,Q_1 + \frac{1}{R_1} + \frac{1}{\frac{1}{(\mathrm{i}\,\omega)^{\alpha_2}\,Q_2} + R_2}}$$

$$Z(\omega) = \frac{R_1\,\left(1 + (\mathrm{i}\,\omega)^{\alpha_2}\,Q_2\,R_2\right)}{1 + (\mathrm{i}\,\omega)^{\alpha_1}\,Q_1\,R_1 + (\mathrm{i}\,\omega)^{\alpha_2}\,Q_2\,R_1 + (\mathrm{i}\,\omega)^{\alpha_2}\,Q_2\,R_2 + (\mathrm{i}\,\omega)^{\alpha_1+\alpha_2}\,Q_1\,Q_2\,R_1\,R_2}$$

$$\begin{aligned}\mathrm{Re}\;Z(\omega) = &(R_1\,(1 + \omega^{\alpha_2}\,Q_2\,(\omega^{\alpha_2}\,Q_2\,R_2\,(R_1 + R_2) + C_{\alpha 2}\,(R_1 + 2\,R_2)) + \\ &\omega^{\alpha_1}\,C_{\alpha 1}\,Q_1\,R_1\,(1 + \omega^{\alpha_2}\,Q_2\,R_2\,(2\,C_{\alpha 2} + \omega^{\alpha_2}\,Q_2\,R_2))))\,/ \\ &(1 + \omega^{\alpha_2}\,Q_2\,(R_1 + R_2)\,(2\,C_{\alpha 2} + \omega^{\alpha_2}\,Q_2\,(R_1 + R_2)) + \\ &\omega^{2\,\alpha_1}\,{Q_1}^2\,{R_1}^2\,(1 + \omega^{\alpha_2}\,Q_2\,R_2\,(2\,C_{\alpha 2} + \omega^{\alpha_2}\,Q_2\,R_2)) + 2\,\omega^{\alpha_1}\,Q_1\,R_1 \\ &\times (C_{\alpha 1} + \omega^{\alpha_2}\,Q_2\,(C_{\alpha 1 m \alpha 2}\,R_1 + 2\,C_{\alpha 1}\,C_{\alpha 2}\,R_2 + \omega^{\alpha_2}\,C_{\alpha 1}\,Q_2\,R_2\,(R_1 + R_2))))\end{aligned}$$

$$\begin{aligned}\mathrm{Im}\;Z(\omega) = &\left({R_1}^2\,(-(\omega^{\alpha_1}\,Q_1\,(1 + \omega^{\alpha_2}\,Q_2\,R_2\,(2\,C_{\alpha 2} + \omega^{\alpha_2}\,Q_2\,R_2))\,S_{\alpha 1}) - \omega^{\alpha_2}\,Q_2\,S_{\alpha 2})\right)/ \\ &(1 + \omega^{\alpha_2}\,Q_2\,(R_1 + R_2)\,(2\,C_{\alpha 2} + \omega^{\alpha_2}\,Q_2\,(R_1 + R_2)) + \\ &\omega^{2\,\alpha_1}\,{Q_1}^2\,{R_1}^2\,(1 + \omega^{\alpha_2}\,Q_2\,R_2\,(2\,C_{\alpha 2} + \omega^{\alpha_2}\,Q_2\,R_2)) + 2\,\omega^{\alpha_1}\,Q_1\,R_1 \\ &\times (C_{\alpha 1} + \omega^{\alpha_2}\,Q_2\,(C_{\alpha 1 m \alpha 2}\,R_1 + 2\,C_{\alpha 1}\,C_{\alpha 2}\,R_2 + \omega^{\alpha_2}\,C_{\alpha 1}\,Q_2\,R_2\,(R_1 + R_2))))\end{aligned}$$

$$Z(\omega) = \frac{R_1\,\left(1 + (\mathrm{i}\,\omega)^{\alpha_2}\,\tau_2\right)}{1 + (\mathrm{i}\,\omega)^{\alpha_1}\,\tau_1 + (1 + R_1/R_2)\,(\mathrm{i}\,\omega)^{\alpha_2}\,\tau_2 + (\mathrm{i}\,\omega)^{\alpha_1+\alpha_2}\,\tau_1\,\tau_2}$$

$$\tau_1 = Q_1\,R_1\;,\;\tau_2 = Q_2\,R_2$$

Bibliography

[1] Brug, G. J., van den Eeden, A. L. G., Sluyters-Rehbach, M., and Sluyters, J. H. The analysis of electrode impedance complicated by the presence of a constant element. *J. Electroanal. Chem. 176* (1984), 275–295.

[2] Chabli, A., Diaco, T., and Diard, J.-P. Détermination de la séquence de pondération d'une pile leclanché à l'aide d'une méthode d'intercorrélation. *J. Applied Electrochem. 11* (1981), 661.

[3] Han, D. G., and Choi, G. M. Computer simulation of the electrical conductivity of composites: the effect of geometrical arrangement. *Solid State Ionics 106* (1998), 71–87.

[4] Holzapfel, M., Martinent, A., Alloin, F., LeGorrec, B., Yazami, R., and Montella, C. First lithiation and charge/discharge cycles of graphite materials, investigated by electrochemical impedance spectroscopy. *J. Electroanal. Chem. 546* (2003), 41–50.

[5] Jansen, R., and Beck, F. *Electrochim. Acta 39* (1994), 921.

[6] Matthew-Esteban, J., and Orazem, M. E. On the application of the Kramers-Kronig relations to evaluate the consistency of electrochemical impedance data. *J. Electrochem. Soc. 138* (1991), 67–76.

[7] Messaoudi, B., Joiret, S., Keddam, M., and Takenouti, H. Anodic behaviour of manganese in alkaline medium. *Electrochim. Acta 46* (2001), 2487–2498.

[8] Orazem, M. E., Shukla, P. S., and Membrino, M. A. Extension of the measurement model approach for deconvolution of underlying distributions for impedance measurements. *Electrochim. Acta 47* (2002), 2027–2034.

[9] Seki, S., Kobayashi, Y., Miyashiro, H., Yamanaka, A., Mita, Y., and Iwahori, T. Degradation mechanism analysis of all solid-state lithium polymer rechargeable batteries. In *IMBL 12 Meeting* (2004), The Electrochemical Society. Abs. 386.

[10] Sluyters-Rehbach, M. Impedance of electrochemical systems: Terminology, nomenclature and representation-Part I: Cells with metal electrodes and liqui solution (IUPAC Recommendations 1994). *Pure & Appl. Chem. 66* (1994), 1831–1891.

[11] Tomczyk, P., and Mosialek, M. Investigation of the oxygen electrode reaction in basic molten carbonates using electrochemical impedance spectroscopy. *Electrochim. Acta 46* (2001), 3023–3032.