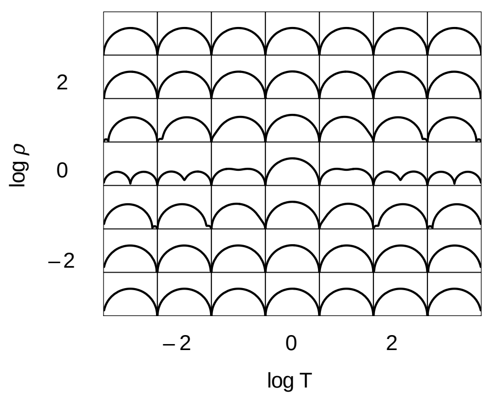
Electrical circuits made of R and C



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January 29, 2001

Contents

Chapter 1

Circuits made of one R and one C

1.1 Circuit (R+C)

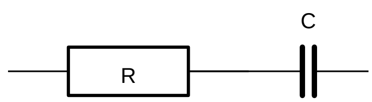

Figure 1.1: Circuit (R+C).

1.1.1 Impedance

$$Z(\omega) = R + \frac{1}{C\, i\, \omega} = \frac{R\,(1 + i\,\omega\,\tau)}{i\,\omega\,\tau}\ ,\ \tau = R\,C$$

$$\mathrm{Re}\ Z(\omega) = R\ ,\ \mathrm{Im}\ Z(\omega) = -\frac{R}{\tau\,\omega}$$

1.1.2 Reduced impedance

$$Z^*(u) = Z/R = 1 + \frac{1}{i\,u} = \frac{1 + i\,u}{i\,u}\ ,\ u = \tau\,\omega\ ,\ \mathrm{Re}\ Z^*(u) = 1\ ,\ \mathrm{Im}\ Z^*(u) = -\frac{1}{u} \tag{1.1}$$

1.2 Circuit (R/C)

1.2.1 Impedance

$$Z(\omega) = \frac{R}{1 + i\,\omega\,\tau}\ ,\ \tau = R\,C$$

$$\mathrm{Re}\ Z(\omega) = \frac{R}{1 + \tau^2\,\omega^2}\ ;\ \mathrm{Im}\ Z(\omega) = -\frac{R\,\tau\,\omega}{1 + \tau^2\,\omega^2}$$

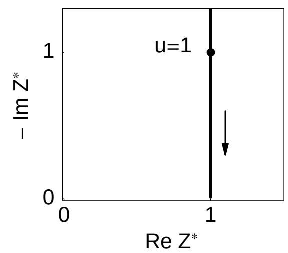

Figure 1.2: Nyquist diagram of the (R+C) circuit reduced impedance (Fig. 1.1, Eq. 1.1).

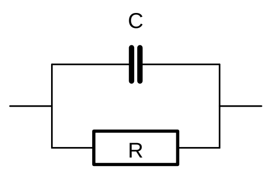

Figure 1.3: Circuit (R/C).

1.2.2 Reduced impedance

$$Z^*(u) = Z/R = \frac{1}{1+i\,u} \; ; \; u = \tau\,\omega \; ; \; \text{Re}\; Z^*(u) = \frac{1}{1+u^2} \; ; \; \text{Im}\; Z^*(u) = -\frac{u}{1+u^2} \tag{1.2}$$

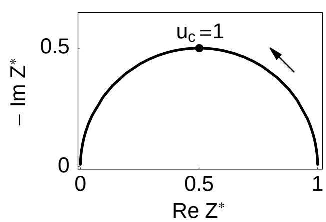

Figure 1.4: Nyquist diagram of the (R/C) circuit reduced impedance (Fig. 1.3, Eq. 1.2).

Chapter 2

Circuits made of two R and one C

2.1 Circuit (R_2+(R_1/C_1))

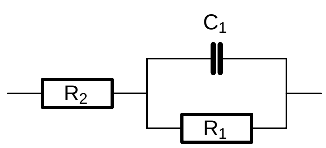

Figure 2.1: Circuit (R_2+(R_1/C_1)).

2.1.1 Impedance

$$Z(\omega) = R_2 + \frac{1}{i\,\omega\,C_1 + \frac{1}{R_1}}$$

$$Z(\omega) = \frac{(R_1 + R_2)\ (1 + i\,\omega\,\tau_2)}{1 + i\,\omega\,\tau_1}\ ,\ \tau_1 = R_1\,C_1\ ,\ \tau_2 = \frac{C_1\,R_1\,R_2}{R_1 + R_2}$$

2.1.2 Reduced impedance

$$Z^*(u) = Z(u)/(R_1 + R_2) = \frac{1 + T\,i\,u}{1 + i\,u} \qquad (2.1)$$

$$u = \tau_1\,\omega\ ,\ T = \tau_2/\tau_1 = R_2/\,(R_1 + R_2) < 1$$

$$\mathrm{Re}\ Z^*(u) = \frac{1 + T\,u^2}{1 + u^2}\ ,\ \mathrm{Im}\ Z^*(u) = \frac{(T-1)\ u}{1 + u^2}$$

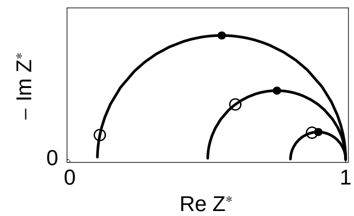

Figure 2.2: Nyquist diagram of the (R_2+(R_1/C_1)) circuit reduced impedance (Fig. 2.1, Eq. 2.1), $T = 0.8, 0.5, 0.1$, semicircle diameter increases for decreasing T). Dots: reduced caracteristic angular frequencies $u_{c1} = 1$; circles: reduced caracteristic angular frequencies $u_{c2} = 1/T$.

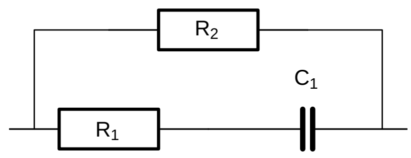

Figure 2.3: Circuit (($R-1$+C_1)/R_2).

2.2 Circuit ((R_1+C_1)/R_2)

2.2.1 Impedance

$$Z(\omega) = \frac{R_2\ (1 + i\,\omega\,\tau_2)}{1 + i\,\omega\,\tau_1}\ ,\ \tau_1 = C_1\ (R_1 + R_2)\ ,\ \tau_2 = R_1\,C_1$$

$$\text{Re}\ Z(\omega) = \frac{R_2\ \left(1 + \omega^2\,\tau_1\,\tau_2\right)}{1 + \omega^2\,{\tau_1}^2}\ ,\ \text{Im}\ Z(\omega) = \frac{\omega\,R_2\ (-\tau_1 + \tau_2)}{1 + \omega^2\,{\tau_1}^2}$$

2.2.2 Reduced impedance (cf. Éq. 2.1)

$$Z^*(u) = Z(u)/R_2 = \frac{1 + T\,i\,u}{1 + i\,u} \tag{2.2}$$

$$u = \tau_1\,\omega\ ,\ T = \tau_2/\tau_1 = R_1/(R_1 + R_2) < 1$$

2.3 Transformation formulas (R+(R/C)) ↔ ((R+C)/R)

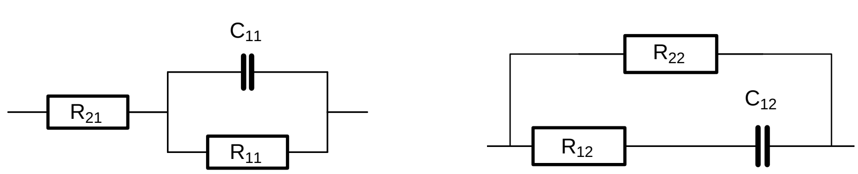

Figure 2.4: (R+(R/C)) and ((R+C)/R circuits are non-distinguishable [3].

2.3.1 Transformation formulas (R+(R/C)) → ((R+C)/R)

$$R_{22} = R_{11} + R_{21}, R_{12} = R_{11} + \frac{{R_{11}}^2}{R_{21}}, C_{12} = \frac{C_{11}\,{R_{21}}^2}{\left(R_{11} + R_{21}\right)^2}$$

2.3.2 Transformation formulas ((R+C)/R) → (R+(R/C))

$$C_{11} = \frac{C_{12}\left(R_{12} + R_{22}\right)^2}{{R_{22}}^2}, R_{11} = \frac{R_{12}\,R_{22}}{R_{12} + R_{22}}, R_{21} = \frac{{R_{22}}^2}{R_{12} + R_{22}}$$

Chapter 3

Circuits made of one R and two C

3.1 Circuit $((R_1/C_1)+C_2)$ (Fig. 3.1)

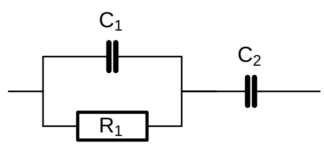

Figure 3.1: Circuit $((R_1/C_1)+C_2)$.

3.1.1 Impedance

$$Z(\omega) = \frac{1}{i\,\omega\,C_1 + \frac{1}{R_1}} + \frac{1}{i\,\omega\,C_2} = \frac{1 + i\,\omega\;(C_1 + C_2)\;R_1}{i\,\omega\,C_2\;(1 + i\,\omega\,C_1\,R_1)}$$

3.1.2 Time constants # 1

$$Z(\omega) = \frac{1 + i\,\omega\,\tau_2}{i\,\omega\,C_2\;(1 + i\,\omega\,\tau_1)}\;,\;\tau_1 = R_1\,C_1\;,\;\tau_2 = (C_1 + C_2)\,R_1\;,\;\tau_1 < \tau_2$$

$$\mathrm{Re}\;Z(\omega) = \frac{-\tau_1 + \tau_2}{C_2 + \omega^2\,C_2\,{\tau_1}^2}\;,\;\mathrm{Im}\;Z(\omega) = -\frac{1 + \omega^2\,\tau_1\,\tau_2}{\omega\,C_2 + \omega^3\,C_2\,{\tau_1}^2}$$

$$\lim_{\omega\to 0}\mathrm{Re}\;Z(\omega) = R_1$$

3.1.3 Reduced impedance # 1

$$Z^*(u) = Z(u)/R_1 = \frac{1 + T\,i\,u}{(T-1)\,i\,u\;(1 + i\,u)} \tag{3.1}$$

$$u = \omega\,\tau_1\;,\;T = \tau_2/\tau_1 = 1 + C_2/C_1 > 1$$

$$\text{Re } Z^*(u) = \frac{1}{1+u^2} \text{ , Im } Z^*(u) = -\frac{1+u^2\, T}{(T-1)\, u\, (1+u^2)}$$

$$\lim_{u \to 0} \text{Re } Z^*(u) = 1$$

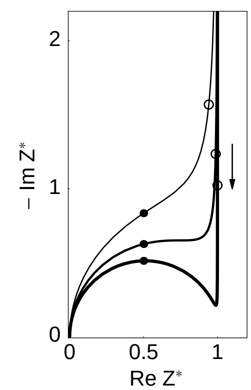

Figure 3.2: Nyquist diagram of the $((R_1/C_1)+C_2)$ circuit reduced impedance (Eq. 3.1, $T = 4, 9, 90$, line thickness increases with increasing T). Horizontal tangent for $T \geq 9$ ($C_2/C_1 \geq 8$) [1]. Dots: reduced caracteristic angular frequencies $u_{c1} = 1$; circles: reduced caracteristic angular frequencies $u_{c2} = 1/T$.

3.1.4 Time constant # 2

$$Z(\omega) = \frac{R_1\ (1 + i\,\omega\,(\tau_1 + \tau_2))}{i\,\omega\ (1 + i\,\omega\,\tau_1)\ \tau_2} \text{ , } \tau_1 = R_1\, C_1 \text{ , } \tau_2 = R_2\, C_2$$

3.1.5 Reduced impedance # 2

$$Z^*(u) = Z(u)/R_1 = \frac{1 + i\,u\,(1+T)}{T\,i\,u\ (1 + i\,u)} \text{ , } u = \omega\,\tau_1 \text{ , } T = \tau_2/\tau_1 = C_2/C_1$$

$$\text{Re } Z^*(u) = \frac{1}{1+u^2} \text{ , Im } Z^*(u) = -\frac{1 + (1+T)\ u^2}{T\,u\ (1+u^2)}$$

$$\lim_{u \to 0} \text{Re } Z^*(u) = 1$$

3.2 Circuit $((R_1+C_2)/C_1)$ (Fig. 3.3)

3.2.1 Impedance

$$Z(\omega) = \frac{1}{i\,\omega\,C_1 + \dfrac{1}{R_1 + \dfrac{1}{i\,\omega\,C_2}}} = \frac{1 + i\,\omega\,C_2\,R_1}{i\,\omega\ (C_1 + C_2)\ \left(1 + \dfrac{i\,\omega\,C_1\,C_2\,R_1}{C_1 + C_2}\right)}$$

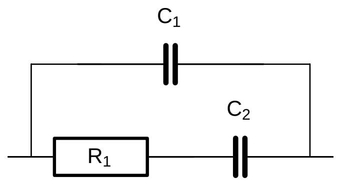

Figure 3.3: Circuit ($(R_1+C_2)/C_1$).

3.2.2 Time constants # 1

$$Z(\omega) = \frac{(1 + i\,\omega\,\tau_2)}{i\,\omega\,(C_1 + C_2)\,(1 + i\,\omega\,\tau_1)}\;,\;\tau_1 = \frac{C_1\,C_2\,R_1}{C_1 + C_2}\;,\;\tau_2 = C_2\,R_1 \qquad (3.2)$$

$$\mathrm{Re}\;Z(\omega) = \frac{-\tau_1 + \tau_2}{(C_1 + C_2)\,(1 + \omega^2\,\tau_1{}^2)}\;,\;\mathrm{Im}\;Z(\omega) = -\frac{1 + \omega^2\,\tau_1\,\tau_2}{\omega\,(C_1 + C_2)\,(1 + \omega^2\,\tau_1{}^2)}$$

$$\lim_{\omega\to 0}\mathrm{Re}\;Z(\omega) = \frac{C_2{}^2\,R_1}{(C_1 + C_2)^2}$$

3.2.3 Reduced impedance # 1

$$Z^*(u) = \frac{(C_1 + C_2)^2}{C_2{}^2\,R_1}\,Z(u) = \frac{1 + T\,i\,u}{(T - 1)\;i\,u\,(1 + i\,u)} \qquad (3.3)$$

$$u = \omega\,\tau_1\;,\;T = \tau_2/\tau_1 = 1 + C_2/C_1 > 1$$

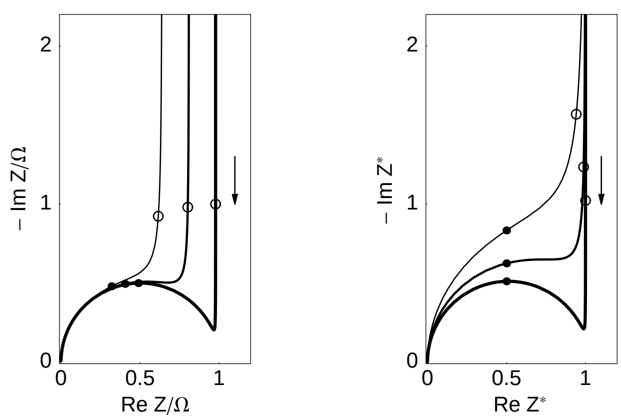

Figure 3.4: Left: Nyquist diagram of the ($(R_1+C_2)/C_1$) circuit impedance (Eq. 3.2, $R_1 = 1\,\Omega$; $C_1 = 10^{-4}$ F cm^{-2}; $C_2/(10^{-4}$F cm$^{-2}) = 4, 9, 90$ *i.e.* $T = 4, 9, 90$). Right: Nyquist diagram of the ($(R_1+C_2)/C_1$) circuit reduced impedance (Eq. 3.3, $T = 4, 9, 90$). Horizontal tangent for $T \geq 9$. ($C_2/C_1 \geq 8$). Line thickness increases with increasing C_2 and T.

3.2.4 Time constants # 2

$$Z(\omega) = \frac{R_1\ (1 + i\,\omega\,\tau_2)}{i\,\omega\ (\tau_2 + \tau_1\ (1 + i\,\omega\,\tau_2))}\ ,\ \tau_1 = R_1\,C_1\ ,\ \tau_2 = R_2\,C_2$$

3.2.5 Reduced impedance # 2

$$Z^*(u) = Z/R_1 = \frac{1 + i\,T\,u}{u\ (i\ (1+T) - T\,u)}\ ,\ u = \omega\,\tau_1\ ,\ T = \tau_2/\tau_1$$

$$\lim_{u\to 0} \mathrm{Re}\ Z(u) = \frac{T^2}{(1+T)^2}$$

3.3 Transformation formulas ((R/C)/C) ↔ ((R+C)/C)

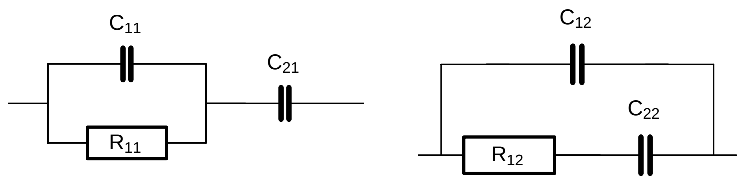

Figure 3.5: Circuit ((R/C)/C) and ((R+C)/C) are non-distinguishable [3].

3.3.1 Transformation formulas ((R/C)/C) → ((R+C)/C)

$$C_{22} = \frac{C_{11}\,C_{21}}{C_{11}+C_{21}}, R_{12} = \frac{(C_{11}+C_{21})^2\,R_{11}}{{C_{21}}^2}, C_{12} = \frac{{C_{21}}^2}{C_{11}+C_{21}}$$

3.3.2 Transformation formulas ((R+C)/C) → ((R/C)/C)

$$C_{11} = C_{22} + \frac{{C_{22}}^2}{C_{12}}, R_{11} = \frac{{C_{12}}^2\,R_{12}}{(C_{12}+C_{22})^2}, C_{21} = C_{12} + C_{22}$$

Chapter 4

Circuits made of two R and two C

4.1 Circuit $((R_1/C_1)+(R_2/C_2))$ (Fig. 4.1)

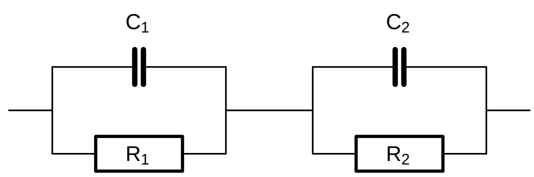

Figure 4.1: Circuit $((R_1/C_1)+(R_2/C_2))$.

4.1.1 Impedance

$$Z(\omega) = \frac{(R_1 + R_2)\ (1 + i\,\omega\,\tau_3)}{(1 + i\,\omega\,\tau_1)\ (1 + i\,\omega\,\tau_2)}$$

$$\tau_1 = R_1\,C_1\ ,\ \tau_2 = R_2\,C_2\ ,\ \tau_3 = \frac{(C_1 + C_2)\ R_1\,R_2}{R_1 + R_2} = \frac{\tau_1\,R_2 + \tau_2\,R_1}{R_1 + R_2}$$

$$\mathrm{Re}\ Z(\omega) = \frac{(R_1 + R_2)\ \left(1 + \omega^2\ (-\tau_1\,\tau_2 + (\tau_1 + \tau_2)\ \tau_3)\right)}{(1 + \omega^2\,{\tau_1}^2)\ (1 + \omega^2\,{\tau_2}^2)}$$

$$\mathrm{Im}\ Z(\omega) = -\frac{\omega\ (R_1 + R_2)\ \left(\tau_1 + \tau_2 + \left(-1 + \omega^2\,\tau_1\,\tau_2\right)\ \tau_3\right)}{(1 + \omega^2\,{\tau_1}^2)\ (1 + \omega^2\,{\tau_2}^2)}$$

4.1.2 Reduced impedance

$$Z^*(u) = Z(u)/\,(R_1 + R_2) = \frac{1 + \rho + (T + \rho)\ i\,u}{(1 + \rho)\,(1 + i\,u)\ (1 + i\,u\,T)}$$

$$u = R_2\,C_2\,\omega\ ,\ \rho = R_1/R_2\ ,\ T = R_1\,C_1/\,(R_2\,C_2) = \gamma\,\rho\ ,\ \gamma = C_1/C_2$$

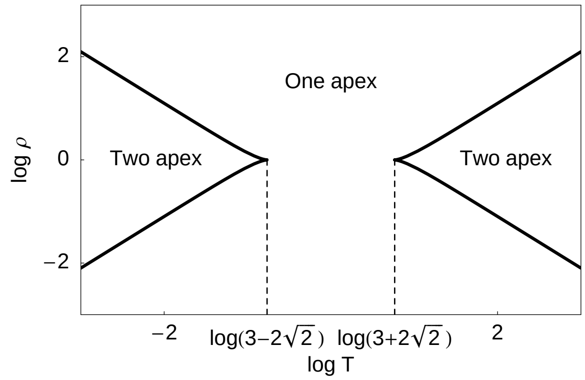

Figure 4.2: Case diagram for the $((R_1/C_1)+(R_2/C_2))$ circuit. $\log\rho$ *vs.* $\log T$ plane.

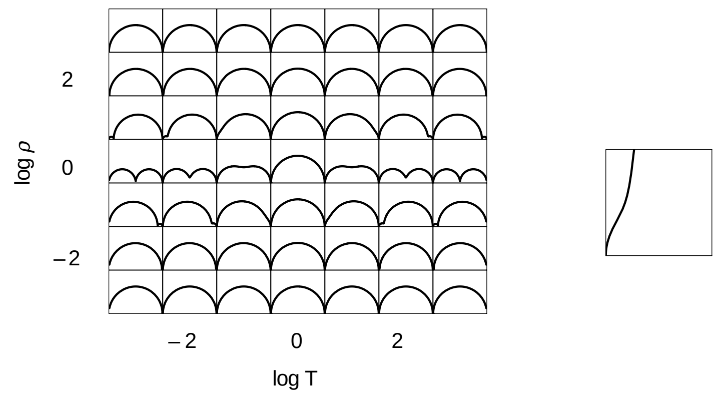

Figure 4.3: Impedance diagrams array for the $((R_1/C_1)+(R_2/C_2))$ circuit ($\log\rho$ *vs.* $\log T$ plane) and enlargment of the hight frequencies part of the diagram calculated for $T = 10^{-2}$ and $\rho = 10^{-2}$.

4.2 Circuit $((R_1+(R_2/C_2))/C_1)$ (Fig. 4.6)

4.2.1 Impedance

$$Z(\omega) = \frac{(R_1 + R_2)\left(1 + \dfrac{i\,\omega\,C_2\,R_1\,R_2}{R_1 + R_2}\right)}{1 + i\,\omega\,\left(C_2\,R_2 + C_1\,\left(R_1 + R_2\right)\right) + (i\,\omega)^2\,C_1\,C_2\,R_1\,R_2}$$

$$Z(\omega) = \frac{(R_1 + R_2)\,(1 + i\,\omega\,\tau_3)}{(1 + i\,\omega\,\tau_1)\,(1 + i\,\omega\,\tau_2)}\ ,\ \tau_3 = \frac{C_2\,R_1\,R_2}{R_1 + R_2}$$

The impedance poles of a circuit made of R and C are real [2].

$$\tau_1 = \frac{C_2\,R_2 + C_1\,\left(R_1 + R_2\right) - \sqrt{-4\,C_1\,C_2\,R_1\,R_2 + \left(C_2\,R_2 + C_1\,\left(R_1 + R_2\right)\right)^2}}{2} \tag{4.1}$$

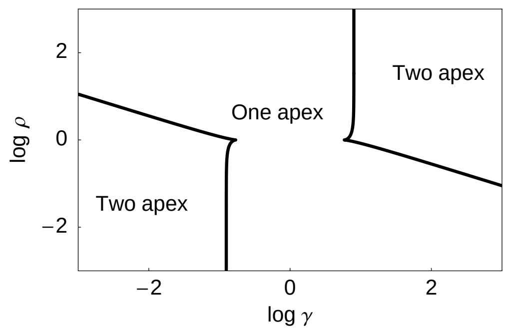

Figure 4.4: Case diagram for the ((R$_1$/C$_1$)+(R$_2$/C$_2$)) circuit. $\log\rho$ *vs.* $\log\gamma$ plane.

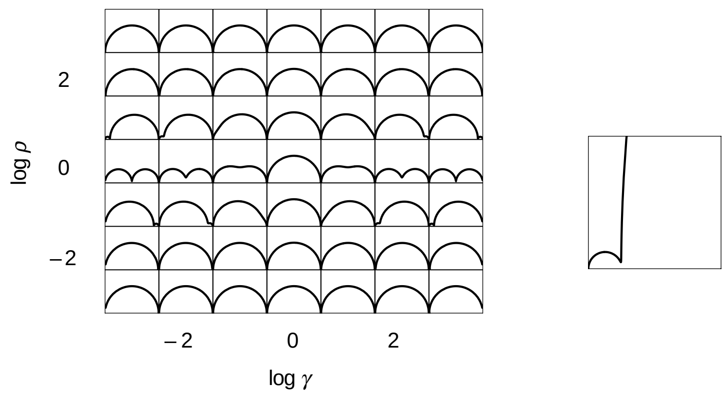

Figure 4.5: Impedance diagrams array for the ((R$_1$/C$_1$)+(R$_2$/C$_2$)) circuit ($\log\rho$ *vs.* $\log\gamma$ plane) and enlargment of the hight frequencies part of the diagram calculated for $\gamma = 10^{-2}$ and $\rho = 10^{-2}$.

$$\tau_2 = \frac{C_2\,R_2 + C_1\,\left(R_1 + R_2\right) + \sqrt{-4\,C_1\,C_2\,R_1\,R_2 + \left(C_2\,R_2 + C_1\,\left(R_1 + R_2\right)\right)^2}}{2} \tag{4.2}$$

4.2.2 Reduced impedance

$$Z^*(u) = Z(u)/\left(R_1 + R_2\right) = \frac{\rho\,\left(1 + \rho + i\,u\,\rho\right)}{\left(1 + \rho\right)\,\left(i\,u\,T + \left(1 + i\,u\right)\,\left(1 + i\,u\,T\right)\,\rho\right)}$$

$$u = R_2\,C_2\,\omega\ ,\ \rho = R_1/R_2\ ,\ T = R_1\,C_1/\left(R_2\,C_2\right) = \gamma\,\rho\ ,\ \gamma = C_1/C_2$$

4.3 Circuit ((C$_1$+(R$_2$/C$_2$))/R$_1$) (Fig. 4.11)

4.3.1 Impedance

$$Z(\omega) = \frac{R_1\,\left(1 + i\,\omega\,\left(C_1 + C_2\right)\,R_2\right)}{1 + i\,\omega\,\left(C_2\,R_2 + C_1\,\left(R_1 + R_2\right)\right) + (i\,\omega)^2\,C_1\,C_2\,R_1\,R_2}$$

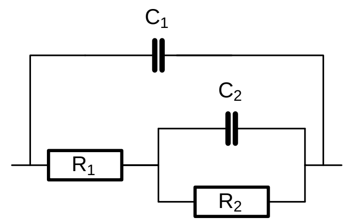

Figure 4.6: Circuit $((R_1+(R_2/C_2))/C_1)$.

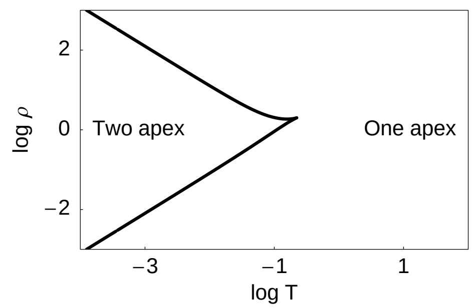

Figure 4.7: Case diagram for the $((R_1+(R_2/C_2))/C_1)$ circuit. $\log \rho$ *vs.* $\log T$ plane.

$$Z(\omega) = \frac{R_1\ (1+i\,\omega\,\tau_3)}{(1+i\,\omega\,\tau_1)\ (1+i\,\omega\,\tau_2)}\ ,\ \tau_3 = (C_1+C_2)\ R_2\ ,\ \tau_1 : \text{Éq. 4.1}\ ,\ \tau_2 : \text{Éq. 4.2}$$

4.3.2 Reduced impedance

$$Z^*(u) = Z(u)/R_1 = \frac{\rho + i\,u\ (\rho+T)}{i\,u\,T + \rho\ (1+i\,u)\ (1+i\,u\,T)}$$

$$u = R_2\,C_2\,\omega\ ,\ \rho = R_1/R_2\ ,\ T = R_1\,C_1/\,(R_2\,C_2) = \gamma\,\rho\ ,\ \gamma = C_1/C_2$$

4.4 Circuit $(((C_2+R_2)/R_1)/C_1)$ (Fig. 4.16)

4.4.1 Impedance

$$Z(\omega) = \frac{R_2\ (1+i\,\omega\,C_1\,R_1)}{1+i\,\omega\ (C_2\,R_2 + C_1\ (R_1+R_2)) + (i\,\omega)^2\,C_1\,C_2\,R_1\,R_2}$$

$$Z(\omega) = \frac{R_2\ (1+i\,\omega\,\tau_3)}{(1+i\,\omega\,\tau_1)\ (1+i\,\omega\,\tau_2)}\ ,\ \tau_3 = C_1\,R_1\ ;\ \tau_1 : \text{Éq. 4.1}\ ,\ \tau_2 : \text{Éq. 4.2}$$

4.4.2 Reduced impedance

$$Z^*(u) = Z(u)/R_1 = \frac{\rho\,(1+i\,u\,T)}{i\,u\,T + \ \rho\,(1+i\,u)\ (1+i\,u\,T)}$$

$$u = R_2\,C_2\,\omega\ ,\ \rho = R_1/R_2\ ,\ T = R_1\,C_1/\,(R_2\,C_2) = \gamma\,\rho\ ,\ \gamma = C_1/C_2$$

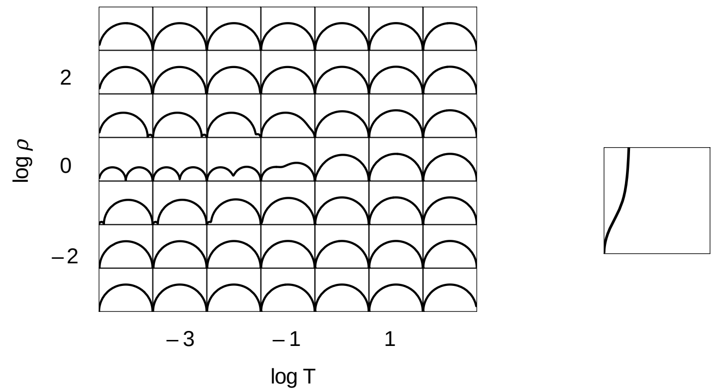

Figure 4.8: Impedance diagrams array for the $((R_1+(R_2/C_2))/C_1)$ circuit circuit ($\log\rho$ *vs.* $\log T$ plane) and enlargment of the hight frequencies part of the diagram calculated for $T = 10^{-3}$ and $\rho = 10^{-3}$.

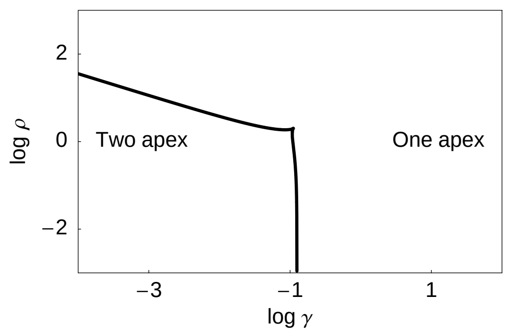

Figure 4.9: Case diagram for the $((R_1+(R_2/C_2))/C_1)$ circuit. $\log\rho$ *vs.* $\log\gamma$ plane.

4.5 Transformation formulas for the four two R and two C circuits

12 transformation formulas exist between the four circuits.

4.5.1 Transformation formulas circuit 2 → circuit 1

$$C_{11} = \frac{1}{2\,C_{22}\,{R_{22}}^2}\sqrt{{C_{22}}^2\,{R_{22}}^2 + 2\,C_{12}\,C_{22}\,R_{22}\,\left(-R_{12}+R_{22}\right) + {C_{12}}^2\,\left(R_{12}+R_{22}\right)^2} \times \left(C_{22}\,R_{22} - C_{12}\,\left(R_{12}+R_{22}\right) + \sqrt{{C_{22}}^2\,{R_{22}}^2 + 2\,C_{12}\,C_{22}\,R_{22}\,\left(-R_{12}+R_{22}\right) + {C_{12}}^2\left(R_{12}+R_{22}\right)^2}\right)$$

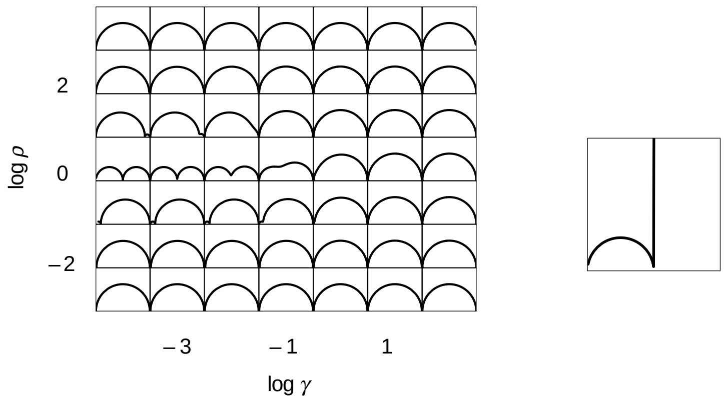

Figure 4.10: Impedance diagrams array for the $((R_1+(R_2/C_2))/C_1)$ circuit circuit ($\log\rho$ *vs.* $\log\gamma$ plane) and enlargment of the hight frequencies part of the diagram calculated for $\gamma = 10^{-3}$ and $\rho = 10^{-3}$.

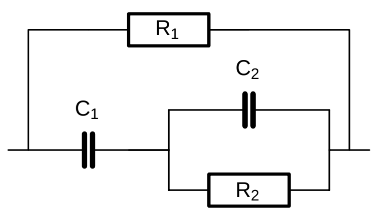

Figure 4.11: Circuit $((C_1+(R_2/C_2))/R_1)$.

$$C_{21} = \frac{1}{2\,C_{22}\,{R_{22}}^2}\sqrt{{C_{22}}^2\,{R_{22}}^2 + 2\,C_{12}\,C_{22}\,R_{22}\,(-R_{12}+R_{22}) + {C_{12}}^2\,(R_{12}+R_{22})^2}$$
$$\times\left(-C_{22}\,R_{22} + C_{12}\,(R_{12}+R_{22}) + \sqrt{{C_{22}}^2\,{R_{22}}^2 + 2\,C_{12}\,C_{22}\,R_{22}\,(-R_{12}+R_{22}) + {C_{12}}^2\,(R_{12}+R_{22})^2}\right)$$

$$R_{21} = \Big(C_{22}\,(R_{12}-R_{22})\,R_{22} - C_{12}\,(R_{12}+R_{22})^2 +$$
$$(R_{12}+R_{22})\,\sqrt{{C_{22}}^2\,{R_{22}}^2 + 2\,C_{12}\,C_{22}\,R_{22}\,(-R_{12}+R_{22}) + {C_{12}}^2\,(R_{12}+R_{22})^2}\Big)\Big/$$
$$\left(2\sqrt{{C_{22}}^2\,{R_{22}}^2 + 2\,C_{12}\,C_{22}\,R_{22}\,(-R_{12}+R_{22}) + {C_{12}}^2\,(R_{12}+R_{22})^2}\right)$$

$$R_{11} = \frac{1}{2}\left(R_{12}+R_{22}+\frac{C_{22}\,R_{22}\,(-R_{12}+R_{22}) + C_{12}\,(R_{12}+R_{22})^2}{\sqrt{{C_{22}}^2\,{R_{22}}^2 + 2\,C_{12}\,C_{22}\,R_{22}\,(-R_{12}+R_{22}) + {C_{12}}^2\,(R_{12}+R_{22})^2}}\right)$$

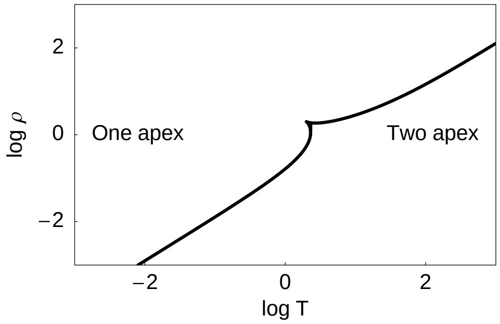

Figure 4.12: Case diagram for the $((C_1+(R_2/C_2))/R_1)$ circuit. $\log\rho$ *vs.* $\log T$ plane.

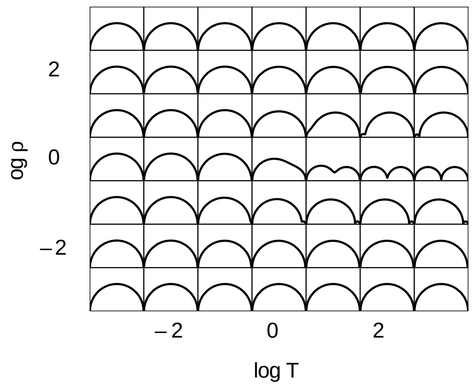

Figure 4.13: Impedance diagrams for the $((C_1+(R_2/C_2))/R_1)$ circuit. $\log\rho$ *vs.* $\log T$ plane.

4.5.2 Transformation formulas circuit 1 → circuit 2

$$C_{12} = \frac{C_{11}\,C_{21}}{C_{11}+C_{21}}, C_{22} = \frac{\left({C_{11}}^2\,R_{11}+{C_{21}}^2\,R_{21}\right)^2}{(C_{11}+C_{21})\,(C_{11}\,R_{11}-C_{21}\,R_{21})^2}$$

$$R_{12} = \frac{(C_{11}+C_{21})^2\,R_{11}\,R_{21}}{{C_{11}}^2\,R_{11}+{C_{21}}^2\,R_{21}}, R_{22} = \frac{(C_{11}\,R_{11}-C_{21}\,R_{21})^2}{{C_{11}}^2\,R_{11}+{C_{21}}^2\,R_{21}}$$

4.5.3 Transformation formulas circuit 3 → circuit 1

$$\begin{aligned}C_{11} = \frac{1}{2\,{C_{13}}^2\,{R_{13}}^2}\Big({C_{13}}^2\,(C_{13}+C_{23})\,{R_{13}}^2+2\,C_{13}\,\left({C_{13}}^2-{C_{23}}^2\right)\,R_{13}\,R_{23}+\\ (C_{13}+C_{23})^3\,{R_{23}}^2-\Big(C_{13}\,(C_{13}-C_{23})\,R_{13}+(C_{13}+C_{23})^2\,R_{23}\Big)\\ \times\sqrt{{C_{23}}^2\,{R_{23}}^2+2\,C_{13}\,C_{23}\,R_{23}\,(-R_{13}+R_{23})+{C_{13}}^2\,(R_{13}+R_{23})^2}\Big)\end{aligned}$$

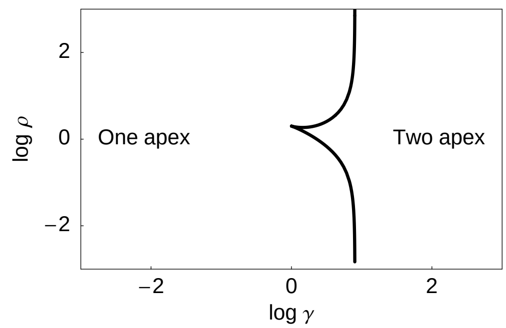

Figure 4.14: Case diagram for the ((C₁+(R₂/C₂))/R₁) circuit. $\log\rho$ *vs.* $\log\gamma$ plane.

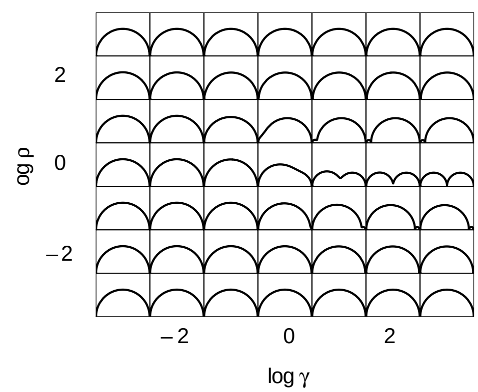

Figure 4.15: Impedance diagrams for the ((C₁+(R₂/C₂))/R₁) circuit. $\log\rho$ *vs.* $\log\gamma$ plane.

$$C_{21} = \frac{1}{2\,{C_{13}}^2\,{R_{13}}^2}\left({C_{13}}^2\,(C_{13}+C_{23})\,{R_{13}}^2 + 2\,C_{13}\,\left({C_{13}}^2-{C_{23}}^2\right)\,R_{13}\,R_{23} + (C_{13}+C_{23})^3\,{R_{23}}^2 + \left(C_{13}\,(C_{13}-C_{23})\,R_{13} + (C_{13}+C_{23})^2\,R_{23}\right) \times\sqrt{{C_{23}}^2\,{R_{23}}^2 + 2\,C_{13}\,C_{23}\,R_{23}\,(-R_{13}+R_{23}) + {C_{13}}^2\,(R_{13}+R_{23})^2}\right)$$

$$R_{21} = R_{13}\,\left(C_{13}\,R_{13} - (C_{13}+C_{23})\,R_{23} + \sqrt{{C_{23}}^2\,{R_{23}}^2 + 2\,C_{13}\,C_{23}\,R_{23}\,(-R_{13}+R_{23}) + {C_{13}}^2\,(R_{13}+R_{23})^2}\right) / \left(2\,\sqrt{{C_{23}}^2\,{R_{23}}^2 + 2\,C_{13}\,C_{23}\,R_{23}\,(-R_{13}+R_{23}) + {C_{13}}^2\,(R_{13}+R_{23})^2}\right)$$

$$R_{11} = \frac{R_{13}}{2}\left(1 + \frac{-C_{13}\,R_{13} + (C_{13}+C_{23})\,R_{23}}{\sqrt{{C_{23}}^2\,{R_{23}}^2 + 2\,C_{13}\,C_{23}\,R_{23}\,(-R_{13}+R_{23}) + {C_{13}}^2\,(R_{13}+R_{23})^2}}\right)$$

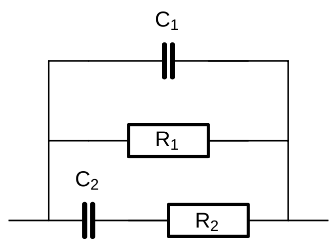

Figure 4.16: Circuit $(((C_2{+}R_2)/R_1)/C_1)$.

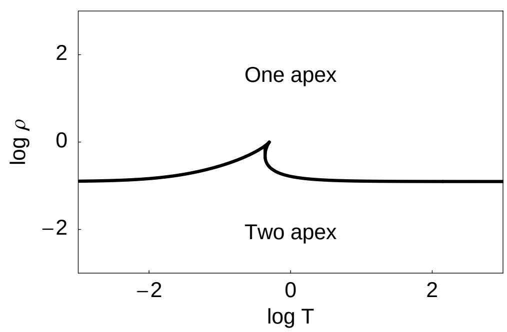

Figure 4.17: Case diagram for the $(((C_2{+}R_2)/R_1)/C_1)$ circuit. $\log\rho$ *vs.* $\log T$ plane.

4.5.4 Transformation formulas circuit 1 → circuit 3

$$R_{13} = R_{11} + R_{21}, C_{23} = \frac{C_{11}\,C_{21}\,\left(C_{11}\,{R_{11}}^2 + C_{21}\,{R_{21}}^2\right)}{\left(C_{11}\,R_{11} - C_{21}\,R_{21}\right)^2}$$

$$C_{13} = \frac{C_{11}\,{R_{11}}^2 + C_{21}\,{R_{21}}^2}{\left(R_{11} + R_{21}\right)^2}, R_{23} = \frac{R_{11}\,R_{21}\,\left(R_{11} + R_{21}\right)\,\left(C_{11}\,R_{11} - C_{21}\,R_{21}\right)^2}{\left(C_{11}\,{R_{11}}^2 + C_{21}\,{R_{21}}^2\right)^2}$$

4.5.5 Transformation formulas circuit 4 → circuit 1

$$C_{11} = \frac{1}{2\,C_{24}\,{R_{14}}^2}\sqrt{{C_{14}}^2\,{R_{14}}^2 + 2\,C_{14}\,C_{24}\,R_{14}\,\left(R_{14} - R_{24}\right) + {C_{24}}^2\left(R_{14} + R_{24}\right)^2}$$
$$\times\,\Big(C_{14}\,R_{14} - C_{24}\,\left(R_{14} + R_{24}\right) +$$
$$\sqrt{{C_{14}}^2\,{R_{14}}^2 + 2\,C_{14}\,C_{24}\,R_{14}\,\left(R_{14} - R_{24}\right) + {C_{24}}^2\left(R_{14} + R_{24}\right)^2}\Big)$$

$$C_{21} = \frac{1}{2\,C_{24}\,{R_{14}}^2}\sqrt{{C_{14}}^2\,{R_{14}}^2 + 2\,C_{14}\,C_{24}\,R_{14}\,\left(R_{14} - R_{24}\right) + {C_{24}}^2\left(R_{14} + R_{24}\right)^2}$$
$$\times\,\Big(-C_{14}\,R_{14} + C_{24}\,\left(R_{14} + R_{24}\right) +$$
$$\sqrt{{C_{14}}^2\,{R_{14}}^2 + 2\,C_{14}\,C_{24}\,R_{14}\,\left(R_{14} - R_{24}\right) + {C_{24}}^2\left(R_{14} + R_{24}\right)^2}\Big)$$

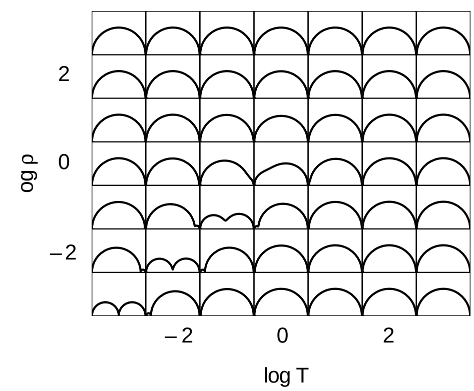

Figure 4.18: Impedance diagrams for the (((C$_2$+R$_2$)/R$_1$)/C$_1$) circuit. $\log\rho$ *vs.* $\log T$ plane.

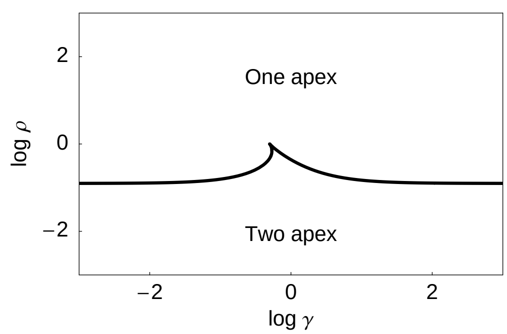

Figure 4.19: Case diagram for the (((C$_2$+R$_2$)/R$_1$)/C$_1$) circuit. $\log\rho$ *vs.* $\log\gamma$ plane.

$$R_{21} = \left(R_{14}\,\left(\left(C_{14}+C_{24}\right)\,R_{14} - C_{24}\,R_{24} + \sqrt{{C_{14}}^2\,{R_{14}}^2 + 2\,C_{14}\,C_{24}\,R_{14}\,\left(R_{14}-R_{24}\right) + {C_{24}}^2\,\left(R_{14}+R_{24}\right)^2}\right)\right) / \left(2\,\sqrt{{C_{14}}^2\,{R_{14}}^2 + 2\,C_{14}\,C_{24}\,R_{14}\,\left(R_{14}-R_{24}\right) + {C_{24}}^2\,\left(R_{14}+R_{24}\right)^2}\right)$$

$$R_{11} = \frac{R_{14}}{2}\,\left(1 + \frac{-\left(C_{14}+C_{24}\right)\,R_{14} + C_{24}\,R_{24}}{\sqrt{{C_{14}}^2\,{R_{14}}^2 + 2\,C_{14}\,C_{24}\,R_{14}\,\left(R_{14}-R_{24}\right) + {C_{24}}^2\,\left(R_{14}+R_{24}\right)^2}}\right)$$

4.5.6 Transformation formulas circuit $1 \to$ circuit 4

$$C_{14} = \frac{C_{11}\,C_{21}}{C_{11}+C_{21}}, R_{14} = R_{11} + R_{21}$$

$$R_{24} = \frac{\left(C_{11}+C_{21}\right)^2\,R_{11}\,R_{21}\,\left(R_{11}+R_{21}\right)}{\left(C_{11}\,R_{11}-C_{21}\,R_{21}\right)^2}, C_{24} = \frac{\left(C_{11}\,R_{11}-C_{21}\,R_{21}\right)^2}{\left(C_{11}+C_{21}\right)\,\left(R_{11}+R_{21}\right)^2}$$

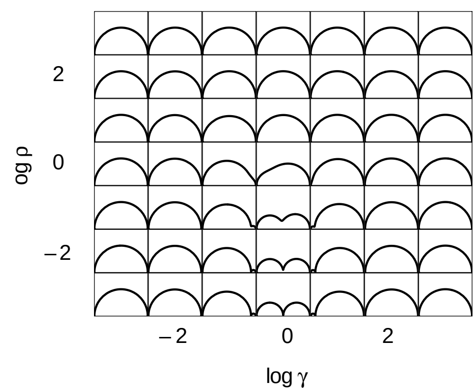

Figure 4.20: Impedance diagrams for the $(((C_2{+}R_2)/R_1)/C_1)$ circuit. $\log\rho$ *vs.* $\log\gamma$ plane.

4.5.7 Transformation formulas circuit $3 \rightarrow$ circuit 2

$$C_{12} = \frac{C_{13}\,C_{23}}{C_{13}+C_{23}}, C_{22} = \frac{\left({C_{13}}^2\,R_{13} + (C_{13}+C_{23})^2\,R_{23}\right)^2}{{C_{13}}^2\,(C_{13}+C_{23})\,{R_{13}}^2}$$

$$R_{12} = \frac{(C_{13}+C_{23})^2\,R_{13}\,R_{23}}{{C_{13}}^2\,R_{13} + (C_{13}+C_{23})^2\,R_{23}}, R_{22} = \frac{{C_{13}}^2\,{R_{13}}^2}{{C_{13}}^2\,R_{13} + (C_{13}+C_{23})^2\,R_{23}}$$

4.5.8 Transformation formulas circuit $2 \rightarrow$ circuit 3

$$R_{13} = R_{12} + R_{22}, C_{23} = \frac{C_{12}\,\left(C_{22}\,{R_{22}}^2 + C_{12}\,(R_{12}+R_{22})^2\right)}{C_{22}\,{R_{22}}^2}$$

$$C_{13} = C_{12} + \frac{C_{22}\,{R_{22}}^2}{(R_{12}+R_{22})^2}, R_{23} = \frac{{C_{22}}^2\,R_{12}\,{R_{22}}^3\,(R_{12}+R_{22})}{\left(C_{22}\,{R_{22}}^2 + C_{12}\,(R_{12}+R_{22})^2\right)^2}$$

4.5.9 Transformation formulas circuit $4 \rightarrow$ circuit 2

$$C_{12} = C_{14}, C_{22} = \frac{C_{24}\,(R_{14}+R_{24})^2}{{R_{14}}^2}, R_{12} = \frac{R_{14}\,R_{24}}{R_{14}+R_{24}}, R_{22} = \frac{{R_{14}}^2}{R_{14}+R_{24}}$$

4.5.10 Transformation formulas circuit $2 \rightarrow$ circuit 4

$$C_{14} = C_{12}, R_{14} = R_{12} + R_{22}, R_{24} = R_{12} + \frac{{R_{12}}^2}{R_{22}}, C_{24} = \frac{C_{22}\,{R_{22}}^2}{(R_{12}+R_{22})^2}$$

4.5.11 Transformation formulas circuit $4 \rightarrow$ circuit 3

$$R_{13} = R_{14}, C_{23} = C_{14} + \frac{{C_{14}}^2}{C_{24}}, C_{13} = C_{14} + C_{24}, R_{23} = \frac{{C_{24}}^2\,R_{24}}{(C_{14}+C_{24})^2}$$

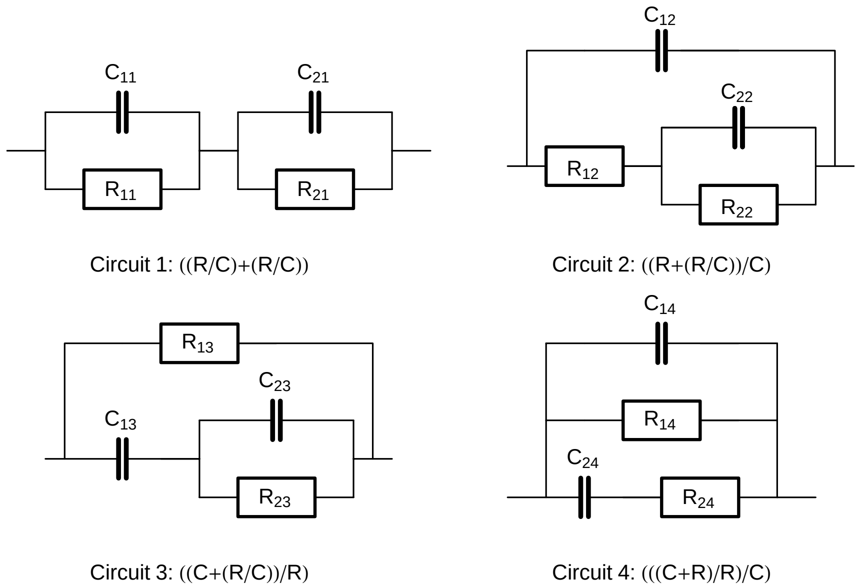

Figure 4.21: The four circuits are non-distinguishable [3].

4.5.12 Transformation formulas circuit $3 \rightarrow$ circuit 4

$$C_{14} = \frac{C_{13}\, C_{23}}{C_{13} + C_{23}}, R_{24} = \frac{(C_{13} + C_{23})^2\, R_{23}}{{C_{13}}^2}, R_{14} = R_{13}, C_{24} = \frac{{C_{13}}^2}{C_{13} + C_{23}}$$

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